1.5 Skalarni umnožak

Definicija 1.10 *Neka su dani vektori $\vec{a} \neq \vec{0}$ i $\vec{b} \neq \vec{0}$ i neka je $\varphi = \measuredangle\left(\overrightarrow{a}, \overrightarrow{b}\right)$. Skalarni umnožak vektora $\vec{a}$ i $\vec{b}$ je realan broj (skalar)*

$$\overrightarrow{a} \cdot \overrightarrow{b} = |\overrightarrow{a}| \cdot \left|\overrightarrow{b}\right| \cdot \cos\varphi.$$

Ako je $\vec{a} = \vec{0}$ ili $\vec{b} = \vec{0}$, onda je $\varphi = \measuredangle\left(\overrightarrow{a}, \overrightarrow{b}\right)$ nije definiran, pa definiramo

$$\overrightarrow{a} \cdot \overrightarrow{b} := 0.$$

Preslikavanje

$$s : V^3 \times V^3 \longrightarrow \mathbb{R}$$

definirano s

$$s\left(\left(\overrightarrow{a}, \overrightarrow{b}\right)\right) = \overrightarrow{a} \cdot \overrightarrow{b},$$

nazivamo skalarno množenje u V^3.

Uočimo:

$$\left(\vec{a}, \vec{b}\right) \in V^3 \times V^3 \xrightarrow{s} \vec{a} \cdot \vec{b} \in \mathbb{R}.$$

Iz definicije neposredno slijedi da je za svaki $\vec{a} \in V^3$

$$\overrightarrow{a} \cdot \overrightarrow{a} = |\overrightarrow{a}|^2 \geq 0.$$

i

$$(-\overrightarrow{a}) \cdot \overrightarrow{a} = -|\overrightarrow{a}|^2.$$

Karakteriziracija okomitost dvaju vektora:

Propozicija 1.9 *Neka su* $\vec{a} \neq \vec{0}$ *i* $\vec{b} \neq \vec{0}$ *bilo koji vektori iz* V^3*. Ti su vektori okomiti ako i samo ako je* $\vec{a} \cdot \vec{b} = 0$.

Dokaz:

Propozicija 1.10 *Neka su* $\vec{a}$ *i* $\vec{b}$ *bilo koji vektori iz* V^3*. Tada vrijedi:*

$$\left(\vec{a} + \vec{b}\right)^2 = \vec{a}^2 + \vec{b}^2 + 2\vec{a} \cdot \vec{b}$$
$$\left(\vec{a} - \vec{b}\right)^2 = \vec{a}^2 + \vec{b}^2 - 2\vec{a} \cdot \vec{b}$$

Dokaz:

Teorem 1.4 *Neka su* $\vec{a}$, $\vec{b}$, $\vec{c}$ *proizvoljni vektori iz* V^3 *i* λ *proizvoljni skalar iz* R*, tada vrijedi:*

1. $\overrightarrow{a} \cdot \overrightarrow{b} = \overrightarrow{b} \cdot \overrightarrow{a}$; *(komutativnost)*

2. $(\lambda\vec{a}) \cdot \overrightarrow{b} = \lambda \left(\overrightarrow{a} \cdot \overrightarrow{b}\right)$; *(kvaziasocijativnost)*

3. $\overrightarrow{a} \cdot \left(\overrightarrow{b} + \vec{c}\right) = \overrightarrow{a} \cdot \overrightarrow{b} + \overrightarrow{a} \cdot \overrightarrow{c}$; *(distributivnost prema zbrajanju)*

4. $\overrightarrow{a}^2 \geq 0$ *i* $\overrightarrow{a}^2 = 0$ *ako i samo ako je* $\overrightarrow{a} = \vec{0}$. *(pozitivna definitnost)*

Dokaz:

Posljedica 1.1 *Neka su* $\vec{a}$*,* $\vec{b}$*,* $\vec{c}$ *proizvoljni vektori iz* V^3 *i* λ *proizvoljni skalar iz* R*, tada vrijedi:*

1. $\vec{a} \cdot \left(\lambda \overrightarrow{b}\right) = \lambda \left(\overrightarrow{a} \cdot \overrightarrow{b}\right);$

2. $\left(\overrightarrow{a} + \overrightarrow{b}\right) \cdot \vec{c} = \overrightarrow{a} \cdot \overrightarrow{c} + \overrightarrow{b} \cdot \overrightarrow{c}.$

Dokaz:

Neka je $B = (\vec{i}, \vec{j}, \vec{k})$ ortonormirana baza u V^3. Tada za koordinatne vektore baze vrijedi ova tablica skalarnih produkata:

$\cdot$	$\vec{i}$	$\vec{j}$	$\vec{k}$
$\vec{i}$	1	0	0
$\vec{j}$	0	1	0
$\vec{k}$	0	0	1

Propozicija 1.11 Neka su $\vec{a} = (\alpha_1, \alpha_2, \alpha_3)$, $\vec{b} = (\beta_1, \beta_2, \beta_3)$ vektori iz V^3 dani svojim pravokutnim koordinatama u bazi $B = (\vec{i}, \vec{j}, \vec{k})$. Tada je

$$\overrightarrow{a} \cdot \overrightarrow{b} = \alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3.$$

Dokaz:

Posljedica 1.2 *Neka su* $\vec{a} = (\alpha_1, \alpha_2, \alpha_3)$*,* $\vec{b} = (\beta_1, \beta_2, \beta_3)$ *vektori iz* V^3 *dani svojim pravokutnim koordinatama u bazi* $B = (\vec{i}, \vec{j}, \vec{k})$. *Tada je:*

1.

$$|\overrightarrow{a}| = \sqrt{\alpha_1^2 + \alpha_2^2 + \alpha_3^2}.$$

2.

$$\cos\measuredangle\left(\overrightarrow{a}, \overrightarrow{b}\right) = \frac{\alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3}{\sqrt{\alpha_1^2 + \alpha_2^2 + \alpha_3^2} \cdot \sqrt{\beta_1^2 + \beta_2^2 + \beta_3^2}}$$

3. *Vektori* $\vec{a}$ *i* $\vec{b}$ *su okomiti ako i samo ako je*

$$\alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3 = 0.$$

Dokaz:

Prikloni kutevi $\varphi_1,\ \varphi_2,\ \varphi_3$ vektora $\vec{a} \neq \vec{0}$ su kutevi koje taj vektor zatvara s vektorima $\vec{i}$, $\vec{j}$ i $\vec{k}$, redom. Ako je $\vec{a} = (\alpha_1, \alpha_2, \alpha_3)$, tada je

$$\cos\varphi_1 = \cos\measuredangle\left(\overrightarrow{a}, \overrightarrow{i}\right) = \frac{\alpha_1}{\sqrt{\alpha_1^2 + \alpha_2^2 + \alpha_3^2}}.$$

$$\cos\varphi_2 = \cos\measuredangle\left(\overrightarrow{a}, \vec{j}\right) = \frac{\alpha_2}{\sqrt{\alpha_1^2 + \alpha_2^2 + \alpha_3^2}},$$

$$\cos\varphi_3 = \cos\measuredangle\left(\overrightarrow{a}, \vec{k}\right) = \frac{\alpha_3}{\sqrt{\alpha_1^2 + \alpha_2^2 + \alpha_3^2}}.$$

Ove skalare nazivamo **kosinusi smjera** vektora $\vec{a}$.

Kosinusi smjera nisu, međutim, neovisni. Vrijedi:

Propozicija 1.12 *Za kosinuse smjera bilo kojeg vektora a vrijedi*

$$\cos^2\varphi_1 + \cos^2\varphi_2 + \cos^2\varphi_2 = 1.$$

<u>Dokaz:</u>

Ako je $\vec{a}$ specijalno jedinični vektor, tj. $|\overrightarrow{a}| = 1$, imamo

$$\cos\varphi_1 = \alpha_1, \quad \cos\varphi_3 = \alpha_2, \quad \cos\varphi_3 = \alpha_3.$$

Dakle, kosinusi smjera vektora $\vec{a}$ jednaki su pravokutnim koordinatama jediničnog vektora $\vec{a}_0 = \frac{\overrightarrow{a}}{|\overrightarrow{a}|}$ u smjeru vektora $\vec{a}$.

1.6 Vektorski umnožak

Definicija 1.11 *Neka su dani nekolinearni vektori* $\vec{a}$ *i* $\vec{b}$ *i neka je* $\varphi = \measuredangle\left(\overrightarrow{a}, \overrightarrow{b}\right)$. *Vektorski umnožak vektora* $\vec{a}$ *i* $\vec{b}$ *je vektor* $\vec{a} \times \vec{b}$ *definiran sa:*

1. $\left|\overrightarrow{a} \times \overrightarrow{b}\right| = |\overrightarrow{a}| \left|\overrightarrow{b}\right| \cdot \sin\varphi$; *(modul)*
2. $\left(\overrightarrow{a} \times \overrightarrow{b}\right) \perp \vec{b}$ *i* $\left(\overrightarrow{a} \times \overrightarrow{b}\right) \perp \overrightarrow{a}$; *(smjer)*
3. *Trojka* $\left(\overrightarrow{a}, \vec{b}, \overrightarrow{a} \times \overrightarrow{b}\right)$ *čini desni koordinatni sustav (orijentacija)*.

Ako su $\vec{a}$ *i* $\vec{b}$ *kolinearni, onda definiramo*

$$\overrightarrow{a} \times \overrightarrow{b} := \vec{0}.$$

Preslikavanje

$$v : V^3 \times V^3 \longrightarrow V^3$$

definirano s

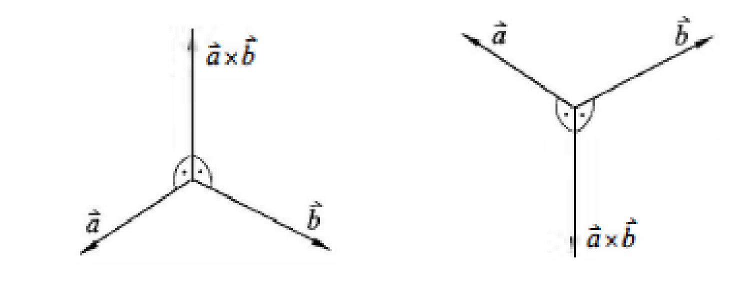
$$v\left(\left(\overrightarrow{a}, \overrightarrow{b}\right)\right) = \overrightarrow{a} \times \overrightarrow{b},$$

nazivamo vektorsko množenje u V^3.

Uočimo:

$$\left(\vec{a}, \vec{b}\right) \in V^3 \times V^3 \xrightarrow{v} \vec{a} \times \vec{b} \in V^3.$$

Vektorski umnožak:



<u>Napomena:</u> Iz definicije vektorskog umnoška slijedi:

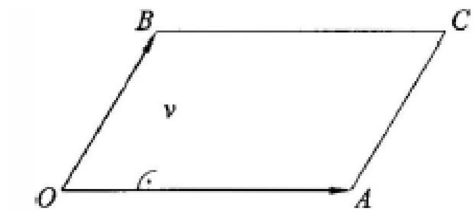
- Ako je $\vec{a}=\vec{0}$ ili $\vec{b}=\vec{0}$, onda su $\vec{a}$ ***i*** $\vec{b}$ kolinearni, pa je $\overrightarrow{a}\times\overrightarrow{b}=\vec{0}$.
- Za sve $\overrightarrow{a}\in V^3$ je $\overrightarrow{a}\times\overrightarrow{a}=\vec{0}$, jer su $\vec{a}$ ***i*** $\overrightarrow{a}$ kolinearni.
- Ako su $\vec{a}$ ***i*** $\vec{b}$ nekolinearni vektori, onda je vektor $\overrightarrow{a}\times\overrightarrow{b}$ "okomit na ravninu određenu" s $\vec{a}$ ***i*** $\vec{b}$.
- Za sve $\overrightarrow{a},\overrightarrow{b}\in V^3$ je $\overrightarrow{a}\times\overrightarrow{b}=-\left(\overrightarrow{b}\times\overrightarrow{a}\right)$; (antikomutativnost)

Propozicija 1.13 $\overrightarrow{a}\times\overrightarrow{b}=\vec{0}$ *ako i samo ako su* $\vec{a}$ ***i*** $\vec{b}$ *kolinearni.*

<u>**Dokaz:**</u>

Geometrijska interpretacija:

Neka su $\vec{a} = \left[\overrightarrow{OA}\right]$ i $\vec{b} = \left[\overrightarrow{OB}\right]$ nekolinearni vektori i neka je $\varphi = \measuredangle\left(\overrightarrow{a}, \overrightarrow{b}\right)$. Norma vektora $\overrightarrow{a} \times \overrightarrow{b}$ je jednaka površini P paralelograma što ga definiraju točke O, A i B



$$\left|\overrightarrow{a} \times \overrightarrow{b}\right| = \left|\overrightarrow{a}\right| \cdot \left|\overrightarrow{b}\right| \cdot \sin\varphi =$$

$$\left|\overline{OA}\right| \left|\overline{OB}\right| \sin\varphi = \left|\overline{OA}\right| v_a = P,$$

gdje je $v_a = \left|\overline{OB}\right| \cdot \sin\varphi = \left|\overrightarrow{b}\right| \cdot \sin\varphi$ visina na stranicu $\overline{OA}$.

Teorem 1.5 *Neka su $\vec{a}$, $\vec{b}$ proizvoljni vektori iz V^3 i λ proizvoljni skalar iz R, tada vrijedi:*

1. $(\lambda\vec{a}) \times \overrightarrow{b} = \lambda\left(\overrightarrow{a} \times \overrightarrow{b}\right)$; *(kvaziasocijativnost)*

2. $\overrightarrow{a} \times \left(\overrightarrow{b} + \vec{c}\right) = \overrightarrow{a} \times \overrightarrow{b} + \overrightarrow{a} \times \overrightarrow{c}$;
(distributivnost prema zbrajanju)

Dokaz:

Posljedica 1.3 *Neka su $\vec{a}$, $\vec{b}$, $\vec{c}$ proizvoljni vektori iz V^3 i λ proizvoljni skalar iz R, tada vrijedi:*

1. $\vec{a} \times \left(\lambda \overrightarrow{b}\right) = \lambda \left(\overrightarrow{a} \times \overrightarrow{b}\right);$

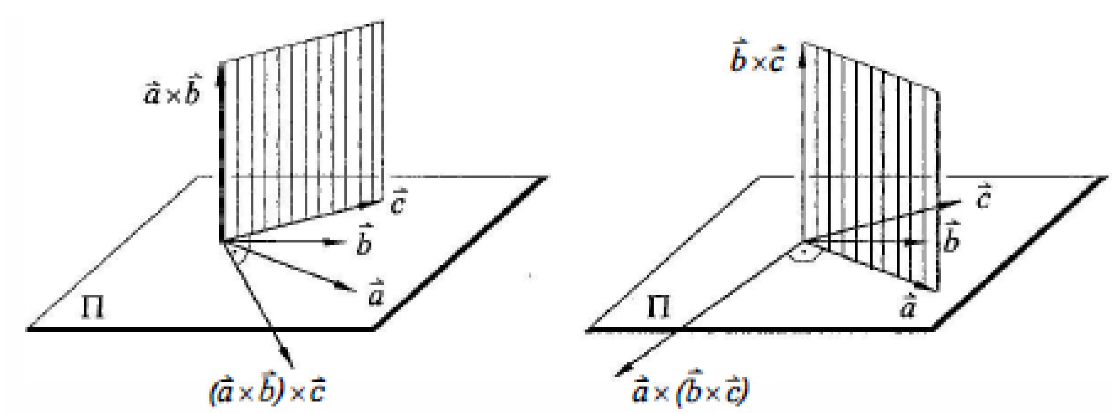
2. $\left(\overrightarrow{a} + \overrightarrow{b}\right) \times \vec{c} = \overrightarrow{a} \times \overrightarrow{c} + \overrightarrow{b} \times \overrightarrow{c}.$

Dokaz:

Napomena:

- Općenito vektorsko množenje nije asocijativno tj. općenito $\left(\overrightarrow{a} \times \overrightarrow{b}\right) \times \vec{c} \neq \overrightarrow{a} \times \left(\overrightarrow{b} \times \vec{c}\right);$

Primjer:Neka su nenul vektori $\overrightarrow{a}, \overrightarrow{b}, \vec{c}$ komplanarni te $\overrightarrow{a}$ i $\vec{c}$ nisu kolinearni:



Preciznije, vrijedi ovaj teorem:

Teorem 1.6 *Neka su $\vec{a}$, $\vec{b}$, $\overrightarrow{c}$ proizvoljni vektori iz V^3, tada vrijedi:*

1. $\left(\overrightarrow{a} \times \overrightarrow{b}\right) \times \vec{c} = (\overrightarrow{a}\vec{c})\overrightarrow{b} - \left(\overrightarrow{b}\vec{c}\right)\overrightarrow{a};$

2. $\overrightarrow{a} \times \left(\overrightarrow{b} \times \vec{c}\right) = (\overrightarrow{a}\vec{c})\overrightarrow{b} - \left(\overrightarrow{a}\overrightarrow{b}\right)\vec{c};$

<u>**Dokaz:**</u>

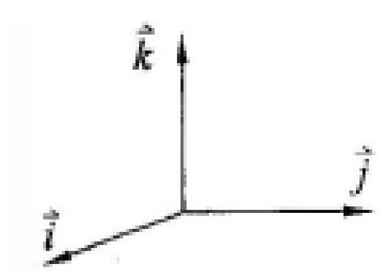
Posljedica 1.4 *Neka su $\vec{a}$, $\vec{b}$, $\vec{c}$ proizvoljni vektori iz V^3, tada vrijedi tzv. Jacobijev identitet:*

$$(\overrightarrow{a} \times \overrightarrow{b}) \times \vec{c} + (\overrightarrow{b} \times \vec{c}) \times \overrightarrow{a} + (\vec{c} \times \overrightarrow{a}) \times \overrightarrow{b} = \vec{0}.$$

<u>**Dokaz:**</u>

<u>Napomena:</u> Iz Jacobijeva identiteta slijedi da su $(\overrightarrow{a} \times \overrightarrow{b}) \times \vec{c}, (\overrightarrow{b} \times \vec{c}) \times \overrightarrow{a}, (\vec{c} \times \overrightarrow{a}) \times \overrightarrow{b}$ komplanarni.

Neka je $B = (\vec{i}, \vec{j}, \vec{k})$ <u>desna</u> ortonormirana baza u V^3



Tada, iz definicije vektorskog umnoška, za koordinatne vektore baze vrijedi:

$$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = \vec{0},$$
$$\vec{i} \times \vec{j} = \vec{k} \Rightarrow \vec{j} \times \vec{i} = -\vec{k}$$
$$\vec{j} \times \vec{k} = \vec{i} \Rightarrow \vec{k} \times \vec{j} = -\vec{i}$$
$$\vec{k} \times \vec{i} = \vec{j} \Rightarrow \vec{i} \times \vec{k} = -\vec{j}$$

ili

$\times$	$\vec{i}$	$\vec{j}$	$\vec{k}$
$\vec{i}$	0	$\vec{k}$	$-\vec{j}$
$\vec{j}$	$-\vec{k}$	0	$\vec{i}$
$\vec{k}$	$\vec{j}$	$-\vec{i}$	0

Propozicija 1.14 Neka su $\vec{a} = (\alpha_1, \alpha_2, \alpha_3)$, $\vec{b} = (\beta_1, \beta_2, \beta_3)$ vektori iz V^3 dani svojim koordinatama u desnoj ortonormiranoj bazi $B = (\vec{i}, \vec{j}, \vec{k})$. Tada je

$$\overrightarrow{a} \times \overrightarrow{b} = (\alpha_2\beta_3 - \alpha_3\beta_2,\ \alpha_3\beta_1 - \alpha_1\beta_3,\ \alpha_1\beta_2 - \alpha_2\beta_1)\,.$$

Dokaz:

Možemo pisati:

$$\overrightarrow{a} \times \overrightarrow{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \end{vmatrix}.$$

1.5 Mješoviti umnožak

Definicija 1.12 *Neka su dani vektori $\vec{a}$, $\vec{b}$* i *$\vec{c}$ i neka je $\varphi = \measuredangle\left(\overrightarrow{a}, \overrightarrow{b}\right)$. Mješoviti umnožak vektora $\vec{a}$, $\vec{b}$* i *$\vec{c}$ je skalar definiran sa:*

$$\left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \vec{c}$$

Oznaka: $\left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \vec{c} := \left[\vec{a}, \vec{b}, \vec{c}\right]$.

Preslikavanje

$$m : V^3 \times V^3 \times V^3 \longrightarrow \mathbb{R}$$

definirano s

$$m\left(\left(\overrightarrow{a}, \overrightarrow{b}, \vec{c}\right)\right) = \left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \vec{c},$$

nazivamo mješovito ili vektorskoskalarno množenje u V^3.

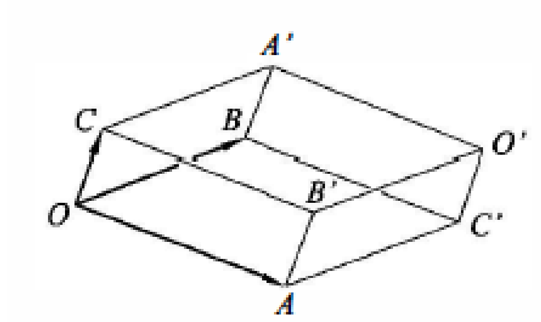
Uočimo:

$$\left(\overrightarrow{a}, \overrightarrow{b}, \vec{c}\right) \in V^3 \times V^3 \times V^3 \xrightarrow{m} \left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \vec{c} = \left[\vec{a}, \vec{b}, \vec{c}\right] \in \mathbb{R}.$$

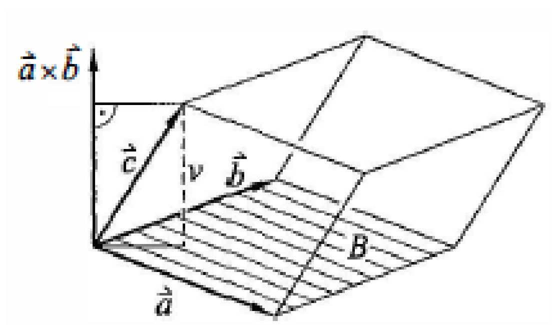
Propozicija 1.15 *Neka su $\vec{a}$, $\vec{b}$ i $\vec{c}$ vektori iz V^3.*
Ti su vektori komplanarni ako i samo ako je $\left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \vec{c} = 0$.

Dokaz:

Geometrijska interpretacija: Neka su $\vec{a} = \left[\overrightarrow{OA}\right]$, $\vec{b} = \left[\overrightarrow{OB}\right]$ i $\vec{c} = \left[\overrightarrow{OC}\right]$ tri nekomplanarna vektora. Točke O, A, B, C definiraju paralelepiped (volum. V)



U tom slučaju kažemo da je paralelepiped "razapet s vektorima $\vec{a}, \vec{b}$ i $\vec{c}$".
Pretpostavimo prvo da je $\left(\vec{a}, \vec{b}, \vec{c}\right)$ desna baza. Tada su $\vec{c}$ i $\overrightarrow{a} \times \overrightarrow{b}$ s iste strane ravnine određene s $\vec{a}$ i $\vec{b}$ (u istom poluprostoru):



Tada je

$$0 \leq \psi = \measuredangle\left(\overrightarrow{a} \times \overrightarrow{b},\ \overrightarrow{c}\right) < \frac{\pi}{2} \quad \Longrightarrow \quad \cos\psi > 0.$$

Onda je visina paralelepipeda na bazu (paralelogram) definiran vektorima $\vec{a}$ i $\vec{b}$ jednaka $v = |\vec{c}| \cdot \cos\psi$.

Sada je

$$\left[\vec{a}, \vec{b}, \vec{c}\right] = \left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \vec{c} = \left|\overrightarrow{a} \times \overrightarrow{b}\right| \cdot |\vec{c}| \cdot \cos\psi = P \cdot v = V$$

Ako je $\left(\vec{a}, \vec{b}, \vec{c}\right)$ lijeva baza. Onda je

$$\frac{\pi}{2} < \psi = \measuredangle\left(\overrightarrow{a} \times \overrightarrow{b}, \ \overrightarrow{c}\right) \leq \pi \quad \Longrightarrow \quad \cos\psi < 0,$$

pa je

$$\left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \vec{c} = \left|\overrightarrow{a} \times \overrightarrow{b}\right| \cdot |\vec{c}| \cdot \cos\psi < 0$$

i

$$V = \left|\overrightarrow{a} \times \overrightarrow{b}\right| \cdot |\vec{c}| \cdot \cos(\pi - \psi) =$$

$$\left|\overrightarrow{a} \times \overrightarrow{b}\right| \cdot |\vec{c}| \cdot (-\cos\psi) = -\left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \vec{c}.$$

Dakle,

$$V = \pm\left[\vec{a}, \vec{b}, \vec{c}\right]$$

gdje predznak $+$ uzimamo ako je $\left(\vec{a}, \vec{b}, \vec{c}\right)$ desna, a $-$ ako je $\left(\vec{a}, \vec{b}, \vec{c}\right)$ lijeva baza.

Uočimo:

$$\left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \vec{c} = 0 \Longleftrightarrow \vec{a},\ \vec{b} \text{ i } \vec{c} \text{ su komplanarni}$$

$$\Longleftrightarrow \text{"degenerirani paralelepiped volumena } = 0\text{"}$$

Propozicija 1.16 *Vrijedi*

$$\left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \vec{c} = \overrightarrow{a} \left(\overrightarrow{b} \times \vec{c}\right).$$

Dokaz:

Posljedica 1.5 *Parnom permutacijom trojke vektora, mješoviti produkt* $\left[\vec{a}, \vec{b}, \vec{c}\right]$ *se ne mijenja, dok neparnom permutacijom mješoviti produkt mijenja samo predznak. Drugim riječima, vrijedi:*

$$\left[\vec{a}, \vec{b}, \vec{c}\right] = \left[\vec{c}, \vec{a}, \vec{b}\right] = \left[\vec{b}, \vec{c}, \vec{a}\right] =$$

$$- \left[\vec{a}, \vec{c}, \vec{b}\right] = - \left[\vec{b}, \vec{a}, \vec{c}\right] = - \left[\vec{c}, \vec{b}, \vec{a}\right]$$

Dokaz:

Propozicija 1.17 *Neka su* $\vec{a} = (\alpha_1, \alpha_2, \alpha_3)$, $\vec{b} = (\beta_1, \beta_2, \beta_3)$, $\vec{c} = (\gamma_1, \gamma_2, \gamma_3)$ *vektori iz* V^3 *dani svojim koordinatama u desnoj ortonormiranoj bazi* $B = (\vec{i}, \vec{j}, \vec{k})$. *Tada je*

$$\left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \vec{c} = \begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix}.$$

Dokaz:

Posljedica 1.6 *Neka su* $\vec{a} = (\alpha_1, \alpha_2, \alpha_3)$, $\vec{b} = (\beta_1, \beta_2, \beta_3)$, $\vec{c} = (\gamma_1, \gamma_2, \gamma_3)$ *vektori iz* V^3 *dani koordinatama u desnoj ortonormiranoj bazi* $B = (\vec{i}, \vec{j}, \vec{k})$.

i) *Ta tri su vektora komplanarna ako i samo ako je*

$$\begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix} = 0.$$

ii) *Ako su ta tri vektora nekomplanarna onda je volumen paralelepipeda razapetog s vektorima* $\vec{a}$, $\vec{b}$ *i* $\vec{c}$ *jednak*

$$V = \pm \begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix}$$