Classical Logic vs. Quantum Logic

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Where to start?

When comparing Classical Logic and Quantum Logic, the first step is defining Classical Logic, or asking:
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What does the expression “Classical Logic” make you think of?
Classical Logic

Classical Logic Properties:

- Binarity,
- Commutativity,
- Distributivity,
- Principle of excluded middle,
- Principle of non-contradiction.
Classical Logic Properties

- **Binarity:**
  Set of truth values - $\mathcal{V}(P) \in \{0, 1\}$

- **Commutativity:**
  $P \cap Q \equiv Q \cap P$, $P \cup Q \equiv Q \cup P$

- **Distributivity:**
  $P \cap (Q \cup R) \equiv (P \cap Q) \cup (P \cap R)$
  $P \cup (Q \cap R) \equiv (P \cup Q) \cap (P \cup R)$

- **Principle of excluded middle:**
  $\mathcal{V}(P \cup \neg P) = 1$

- **Principle of non-contradiction:**
  $\mathcal{V}(\neg (P \cap \neg P)) = 1$, alternatively $\mathcal{V}(P \cap \neg P) = 0$
Quantum Logic Definition

Two main types:

- Birkoff-von Neumann (B-vN) Quantum Logic
- Fuzzy Quantum Logic
Birkhoff-von Neumann (B-vN) Quantum Logic

In 1936 Birkhoff and von Neumann wrote the article “The Logic of Quantum Mechanics”.

Birkhoff and von Neumann wanted to find the logical structure in quantum mechanics which did not conform to classical logic.

B-vN quantum logic is, among other things, a binary, non-distributive and non-commutative lattice.
Jaroslaw Pykacz used the fuzzy sets idea to build Quantum Logic.

Fuzzy Quantum Logic is, defined by Pykacz, partial and infinite - valued (\(\forall (P) \in [0, 1]\)) which is the connection to a probabilistic interpretation of quantum mechanics.

With Fuzzy Quantum Logic we can analyze non - tested experimental sentences.

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There are generally two sets of set operations used in Fuzzy Quantum Logic:
- Zadeh’s set operations
- Giles’ set operations
Lotfi A. Zadeh developed the fuzzy set theory.
Zadeh’s set operations are standard fuzzy sets operations.
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Zadeh’s set operations are standard fuzzy sets operations.

- Complement: $\mathcal{V}(P') = 1 - \mathcal{V}(P)$
- Intersection: $\mathcal{V}(P \cap Q) = \min[\mathcal{V}(P), \mathcal{V}(Q)]$
- Union: $\mathcal{V}(P \cup Q) = \max[\mathcal{V}(P), \mathcal{V}(Q)]$
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- Complement: $\mathcal{V}(P^\prime) = 1 - \mathcal{V}(P)$
- Intersection: $\mathcal{V}(P \cap Q) = \max[\mathcal{V}(P) + \mathcal{V}(Q) - 1, 0]$
- Union: $\mathcal{V}(P \cup Q) = \min[\mathcal{V}(P) + \mathcal{V}(Q), 1]$
Tableau of Comparison

Let’s compare different logics considering given properties.

<table>
<thead>
<tr>
<th>Properties \ Logic</th>
<th>CL</th>
<th>B-vN</th>
<th>Z’sFL</th>
<th>Gs’FL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binarity</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Commutativity</td>
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<td>×</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td>Distributivity</td>
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<td>×</td>
<td>✓</td>
<td>×</td>
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<td>Excluded Middle</td>
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<td>✓</td>
<td>×</td>
<td>✓</td>
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<tr>
<td>Non-Contradiction</td>
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<td>✓</td>
<td>×</td>
<td>✓</td>
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</table>
Distributivity or Excluded Middle?

So the question is:
So the question is:

Which one is more important for quantum mechanics: Distributivity or Excluded Middle?