ONE $(96, 20, 4)$-SYMMETRIC DESIGN AND RELATED NONABELIAN DIFFERENCE SETS

ANKA GOLEMAC, JOŠKO MANDIĆ, AND TANJA VUČIČIĆ

Abstract. New $(96, 20, 4)$-symmetric design has been constructed, unique under the assumption of an automorphism group of order 576 action. The correspondence between a $(96, 20, 4)$-symmetric design having regular automorphism group and a difference set with the same parameters has been used to obtain difference sets in five nonabelian groups of order 96. None of them belongs to the class of groups that allow the application of so far known construction (McFarland, Dillon) for McFarland difference sets.

1. Introductory definitions and assertions

A *symmetric block design* with parameters (v, k, λ) is a finite incidence structure $\mathcal{D} = (\mathcal{V},\mathcal{B},\mathcal{I})$ consisting of $|\mathcal{V}| = v$ points and $|\mathcal{B}| = v$ blocks, where each block is incident with k points and any two distinct points are incident with exactly λ common blocks. An *automorphism* of a symmetric block design $\mathcal{D}$ is a permutation on $\mathcal{V}$ which sends blocks to blocks. The set of all automorphisms of $\mathcal{D}$ forms its full automorphism group denoted by $Aut\mathcal{D}$.

A (v, k, λ) *difference set* is a k-subset $\Delta \subseteq \Gamma$ in a group Γ of order v provided that the multiset of "differences" $\{xy^{-1} \mid x, y \in \Delta, x \neq y\}$ contains each nonidentity element of Γ exactly λ times. A difference set is called *abelian* (*cyclic, nonabelian*) if Γ has the respective property. The *development* of a difference set $\Delta \subseteq \Gamma$ is the incidence structure $dev\Delta = (\Gamma, \{\Delta g \mid g \in \Gamma\}, \in)$.

The following theorem (for the proof see [1], p. 299) refers to the close relation between difference sets and symmetric block designs.

Theorem 1.1. *Let Γ be a finite group of order v and Δ a proper, non-empty k-subset of Γ. Then Δ is a (v, k, λ) difference set in Γ if and only if $dev\Delta$ is a symmetric (v, k, λ) design on which Γ acts regularly (by right multiplication).*

An automorphism $\varphi \in Aut\Gamma$ is called a *multiplier* of Δ if $\varphi(\Delta) = g_1\Delta g_2$ for some $g_1, g_2 \in \Gamma$. When $\varphi(\Delta) = \Delta g$ for some $g \in \Gamma$, φ is called a *right multiplier.* All multipliers of Δ form a group with the subgroup of right multipliers. Up to isomorphism the latter group is determined by the following theorem ([1], p.310).

Theorem 1.2. *Let Δ be a difference set in a group Γ and let M denote the group of all right multipliers of Δ. Then, $M \cong N(\Gamma)/\Gamma$, where $N(\Gamma)$ is the normalizer of Γ in $Autdev\Delta$.*

Obviously, any right multiplier of Δ is an automorphism of the design $dev\Delta$.

Two difference sets Δ^1 (in Γ^1) and Δ^2 (in Γ^2) are *isomorphic* if the designs $dev\Delta^1$ and $dev\Delta^2$ are isomorphic; Δ^1 and Δ^2 are *equivalent* if there exists a group

1991 *Mathematics Subject Classification.* 05B05.

Key words and phrases. Symmetric design, difference set, automorphism group.

isomorphism $\varphi : \Gamma^1 \to \Gamma^2$ such that $\varphi(\Delta^1) = \Delta^2 g$ for a suitable $g \in \Gamma^2$. It is clear that equivalent difference sets Δ^1 and Δ^2 give rise to isomorphic symmetric designs $dev\Delta^1$ and $dev\Delta^2$.

2. More preliminaries

$(96, 20, 4)$-symmetric designs belong to the series with parameters

$$v = q^{d+1}\left(1 + \frac{q^{d+1} - 1}{q - 1}\right), k = q^d \frac{q^{d+1} - 1}{q - 1} \text{ and } \lambda = q^d \frac{q^d - 1}{q - 1}, \tag{1}$$

where q is any prime power and d is any positive integer ([1], p.982 with $q = 4$ and $d = 1$). The series was constructed in two different ways. Wallis [12] gave the construction based on the existence of affine designs. Later, construction due to McFarland [9] was given through difference sets in groups G of order v and of the form $G = E \times K$, where E denotes the elementary abelian group of order q^{d+1} and K is an arbitrary group. Difference sets with parameters (1) are called Mc Farland difference sets. Very important generalization of the result of McFarland was given by Dillon, [5]. He proved McFarland's construction to work out for any group G of order v which contains an elementary abelian subgroup of order q^{d+1} in its center.

Using the well-known coset enumeration method [8], here we perform a construction of one new $(96, 20, 4)$-symmetric design ([1], [3], [4]) under the assumption of a large automorphism group acting transitively on it. The fact that its full automorphism group contains subgroups acting regularly on the design leads to the construction of difference sets in five nonabelian groups of order 96. Checking upon the centers of these groups reveal that they do not satisfy the necessary condition for the appliance of the construction by Dillon.

We mostly employ the terminology and notation of [1]. The examples with detailed construction of difference sets from symmetric designs having regular automorphism group can be found in [11] and [7].

3. Construction of the design

For a starter let us give a brief summary of the coset enumeration method that we use for symmetric design construction, [8].

Let $G \leq Aut\mathcal{D}$ be a group acting on a symmetric design $\mathcal{D}$ defined in Section 1. Then G-orbits on $\mathcal{V}$ denoted by $P^1, \ldots, P^m$ and G-orbits on $\mathcal{B}$ denoted by $B^1, \ldots, B^m$ form a *tactical decomposition* of $\mathcal{D}$, [2]. The construction of $\mathcal{D}$ begins with determining the parameters $\left|P^j\right| = p_j$ and $\left|B^i\right| = b_i$, $j, i \in \{1, \ldots, m\}$ of such a decomposition. In general, as possible b_i and p_j we take into consideration the divisors of $|G|$ for which equations $\sum_{i=1}^m b_i = \sum_{j=1}^m p_j = v$ hold. Specific group action puts further restrictions upon these numbers.

Next, we have to find all possible distributions of the point set $\mathcal{V}$ on the block set $\mathcal{B}$, respecting their selected partitions. These distributions are represented by matrices $T = [\rho_{ij}]$ called *orbit matrices* or *orbit structures*, where ρ_{ij} is the number of points from the orbit P^j incident with any block from the orbit B^i. Entries in T must satisfy the following well known relations:

$$\sum_{j=1}^{m} \rho_{ij} = k, \quad i = 1, \ldots, m;$$

$$\sum_{j=1}^{m} \frac{1}{p_j} \rho_{ij}^2 = \lambda + \frac{k - \lambda}{b_i}, \quad i = 1, \ldots, m \text{ and}$$

$$\sum_{j=1}^{m} \frac{1}{p_j} \rho_{ij} \rho_{lj} = \lambda, \quad i, l = 1, \ldots, m, \, i \neq l.$$

The final step is to find precisely which ρ_{ij} points from the point orbit $P^j = \left\{P_1^j, \ldots, P_{p_j}^j\right\}$ lie on particular block from the block orbit B^i. This procedure is called *indexing* because we usually identify the selected points with their indices from the set $\{1, 2, \ldots, p_j\}$ (and represent G as a permutation group on this set). Indexing has to be performed in such a manner that λ-balancing is achieved. For each orbit $B^i, 1 \leq i \leq m$, it is enough to index a representative block, minding the fact that B^i is stabilized by a subgroup $G^i \leq G$ (unique up to conjugation), $\left|G : G^i\right| = b_i$. The other B^i-blocks are then obtained as G-images of the constructed representative. Indexing all B^i-block representatives $(1 \leq i \leq m)$ concludes the construction.

Now let's specify G to be a group of order 576, in terms of generators and relations given as

$$(2) \qquad \begin{aligned} G = \quad & \langle a, b, c, d, e, f \mid a^4 = b^4 = [a, b] = c^2 = [a, c] = [b, c] = d^3 = 1, \\ & [a, d] = [b, d] = (cd)^2 = e^3 = 1, e^{-1}ae = b, e^{-1}be = a^3b^3, \\ & [c, e] = [d, e] = 1, f^2 = (af)^2 = (bf)^2 = [c, f] = (df)^2 = [e, f] = 1\rangle. \end{aligned}$$

We consider G-action on $(96, 20, 4)$-symmetric design, hereafter denoted by D, so that its subgroup $G_1 = \langle a, b, c, d\rangle \leq G$ of order 96 acts transitively on D, namely in one orbit of length 96 stabilized by subgroup $\langle e, f\rangle \cong Z_6$. To this action corresponds a unique orbit structure of order $m = 1, \rho_{11} = 20$. Thus for our construction we need a permutation representation of the G -generators of degee 96. The one used here is provided by J. Hrabě de Angelis computer program and given in Table I.

TABLE I. The Generating Permutations (Degree 96)

generator a –> (no fixed point)
(1 2 3 4)(5 11 12 13)(6 24 26 32)(7 25 35 20)(8 14 27 41)(9 16 29 50)
(10 17 30 42)(15 21 22 40)(18 31 51 62)(19 53 54 38)(23 46 71 48)
(28 49 52 66)(33 64 72 37)(34 65 76 84)(36 43 44 45)(39 67 78 77)
(47 96 88 81)(55 70 86 56)(57 82 73 69)(58 63 59 61)(60 93 87 85)
(68 80 89 94)(74 79 91 83)(75 95 90 92);

generator b –> (no fixed point)
(1 5 6 7)(2 11 24 25)(3 12 26 35)(4 13 32 20)(8 15 28 33)(9 38 58 67)
(10 36 56 34)(14 21 49 64)(16 19 63 78)(17 43 55 65)(18 46 47 80)
(22 52 72 27)(23 81 68 62)(29 53 59 77)(30 44 70 76)(31 71 96 89)
(37 41 40 66)(39 50 54 61)(42 45 86 84)(48 88 94 51)(57 74 95 60)
(69 83 75 85)(73 91 92 87)(79 90 93 82);

generator c –> (no fixed point)
(1 8)(2 14)(3 27)(4 41)(5 15)(6 28)(7 33)(9 18)(10 82)(11 21)(12 22)(13 40)
(16 31)(17 73)(19 71)(20 37)(23 54)(24 49)(25 64)(26 52)(29 51)(30 69)
(32 66)(34 93)(35 72)(36 79)(38 46)(39 68)(42 57)(43 91)(44 83)(45 74)
(47 58)(48 53)(50 62)(55 92)(56 90)(59 88)(60 84)(61 81)(63 96)(65 87)
(67 80)(70 75)(76 85)(77 94)(78 89)(86 95);

generator d –> (no fixed point)
(1 9 10)(2 16 17)(3 29 30)(4 50 42)(5 38 36)(6 58 56)(7 67 34)(8 82 18)
(11 19 43)(12 53 44)(13 54 45)(14 73 31)(15 79 46)(20 39 84)(21 91 71)

(22 83 48)(23 40 74)(24 63 55)(25 78 65)(26 59 70)(27 69 51)(28 90 47)
(32 61 86)(33 93 80)(35 77 76)(37 60 68)(41 57 62)(49 92 96)(52 75 88)
(64 87 89)(66 95 81)(72 85 94);

generator e –> (6 fixed points)
(2 5 20)(3 6 26)(4 7 11)(12 13 32)(14 15 37)(16 38 39)(17 36 84)(19 50 67)
(21 41 33)(22 40 66)(23 81 48)(24 35 25)(27 28 52)(29 58 59)(30 56 70)
(31 46 68)(34 43 42)(44 45 86)(47 88 51)(49 72 64)(53 54 61)(55 76 65)
(57 93 91)(60 73 79)(62 80 71)(63 77 78)(69 90 75)(74 95 83)(85 87 92)
(89 96 94);

generator f –> (8 fixed points)
(2 4)(5 7)(9 10)(11 20)(12 35)(13 25)(14 41)(15 33)(16 42)(17 50)(18 82)
(19 84)(21 37)(22 72)(23 87)(24 32)(29 30)(31 57)(34 38)(36 67)(39 43)
(40 64)(44 77)(45 78)(46 93)(47 90)(48 85)(49 66)(51 69)(53 76)(54 65)
(55 61)(56 58)(59 70)(60 71)(62 73)(63 86)(68 91)(74 89)(75 88)(79 80)
(81 92)(83 94)(95 96); ¤

Let $V = \{1, 2, \ldots, 96\}$ be the set of points of our design D. As a D-representative block we take a subgroup $\langle e, f\rangle \leq G$ stabilized one. Therefore, it is to be composed from $\langle e, f\rangle$-point orbits as a whole. Using Table I these orbits are easily obtained to be:
{1},{8},{9,10},{18,82},{3,6,26},{27,28,52},{14,15,21,33,37,41},
{16,34,38,39,42,43},{17,19,36,50,67,84},{2,4,5,7,11,20},{22,40,49,64,66,72},
{23,48,81,85,87,92},{12,13,24,25,32,35},{29,30,56,58,59,70},{31,46,57,68,91,93},
{44,45,63,77,78,86},{47,51,69,75,88,90},{53,54,55,61,65,76},{60,62,71,73,79,80},
and {74,83,89,94,95,96}.

There are exactly 1729 possibilities for composing (indexing) 20 points of a possible D-base block out of them. All the possibilities can be λ-balance tested in a short computer time and inappropriate combinations rejected. The resulting representative blocks give rise to symmetric designs which are then checked upon isomorphism with the help of program Nauty [10]. Because all the designs prove to be mutually isomorphic, the final result of the described construction procedure (up to isomorphism) is one selfdual symmetric design which we present by its base block:

(3) $D:$ 1,3,6,9,10,16,18,23,26,34,38,39,42,43,48,81,82,85,87,92

The other blocks of the design can be obtained by producing all $\langle a, b, c, d\rangle$-images of the block (3). The order $\mid AutD \mid = 576$ of the full automorphism group of D is discerned from the output of Nauty as well. Consequently, $AutD \cong G$ and we can sumarize the foregoing in the following theorem.

Theorem 3.1. *There is exactly one* $(96, 20, 4)$*-symmetric design with the automorphism group* $G = \langle a, b, c, d, e, f\rangle$ *of order 576, given by (2), acting on the design so that the subgroup* $\langle a, b, c, d\rangle \leq G$ *acts regularly on it.* $AutD \cong G$.

Remark 3.1. *Design D is not isomorphic to any of the six symmetric* $(96, 20, 4)$ *designs considered in* [3]*, Section 7, since AutD is not isomorphic to any of them corresponding full automorphism groups. In* [3]*, the only design having full automorphism group of order 576 is the one denoted by E. It is known that AutE has* *abelian subgroup of order 96 acting regularly on E. On the other hand, all regular subgroups of $AutD$ are nonabelian; this we show in Section 4. We thank the referee for the suggestion to turn our attention to the results presented in* [3].

4. Construction of difference sets

Aiming to produce difference set with parameters $(96, 20, 4)$ we checked all the subgroups of $G \cong AutD$ of order 96 (seven up to isomophism!) upon the regularity of their action on D. It turned out that the required property of regular action on D had exactly five of them: G_1 and, say, $G_2, \ldots, G_5$. We find these groups to be isomorphic to the following nonabelian groups, respectively:

$$\begin{aligned}
H_1 = \;& \langle x, y, z, w \mid x^4 = y^4 = [x, y] = 1, z^3 = [x, z] = [y, z] = 1, \\
& \quad w^2 = [x, w] = [y, w] = (zw)^2 = 1 \rangle, \\
H_2 = \;& \langle x, y, z, w \mid x^4 = y^4 = [x, y] = 1, z^3 = 1, z^{-1}xz = y, z^{-1}yz = x^3y^3, \\
& \quad w^2 = (xw)^2 = (yw)^2 = [z, w] = 1 \rangle, \\
H_3 = \;& \langle x, y, z, w \mid x^4 = y^4 = [x, y] = 1, z^3 = [x, z] = 1, y^{-1}zy = z^2, \\
& \quad w^2 = (xw)^2 = (yw)^2 = [z, w] = 1 \rangle, \\
H_4 = \;& \langle x, y, z, w \mid x^4 = y^4 = [x, y] = 1, z^3 = [x, z] = [y, z] = 1, \\
& \quad w^2 = (xw)^2 = (yw)^2 = [z, w] = 1 \rangle, \text{ and} \\
H_5 = \;& \langle x, z, w, t, s \mid x^4 = z^3 = [x, z] = 1, w^2 = [x, w] = (zw)^2 = 1, \\
& \quad t^2 = (xt)^2 = (zt)^2 = [w, t] = 1, \\
& \quad s^2 = [x, s] = [z, s] = [w, s] = [t, s] = 1 \rangle.
\end{aligned}$$

Corresponding isomorphisms $\omega_i : H_i \to G_i \leq G, i = 1, \ldots, 5$, are defined by the relations:

$\omega_1(x) = a, \omega_1(y) = b, \omega_1(z) = d, \omega_1(w) = c,$

$\omega_2(x) = a, \omega_2(y) = b, \omega_2(z) = de, \omega_2(w) = cf,$

$\omega_3(x) = a, \omega_3(y) = bc, \omega_3(z) = d, \omega_3(w) = cf,$

$\omega_4(x) = a, \omega_4(y) = b, \omega_4(z) = d, \omega_4(w) = cf,$ and

$\omega_5(x) = a, \omega_5(z) = d, \omega_5(w) = c, \omega_5(t) = g_1, \omega_5(s) = g_2.$

The representations:

$cf =$ (1 8)(2 41)(3 27)(4 14)(5 33)(6 28)(7 15)(9 82)(10 18)
(11 37)(12 72)(13 64)(16 57)(17 62)(19 60)(20 21)(22 35)(23 65)
(24 66)(25 40)(26 52)(29 69)(30 51)(31 42)(32 49)(34 46)(36 80)
(38 93)(39 91)(43 68)(44 94)(45 89)(47 56)(48 76)(50 73)(53 85)
(54 87)(55 81)(58 90)(59 75)(61 92)(63 95)(67 79)(70 88)(71 84)
(74 78)(77 83)(86 96),

$de =$(1 9 10)(2 38 84)(3 58 70)(4 67 43)(5 39 17)(6 59 30)
(7 19 42)(8 82 18)(11 50 34)(12 54 86)(13 61 44)(14 79 68)
(15 60 31)(16 36 20)(21 57 80)(22 74 81)(23 66 83)(24 77 65)
(25 63 76)(26 29 56)(27 90 88)(28 75 51)(32 53 45)(33 91 62)
(35 78 55)(37 73 46)(40 95 48)(41 93 71)(47 52 69)(49 85 89)
(64 92 94)(72 87 96), and

$bc =$(1 15 6 33)(2 21 24 64)(3 22 26 72)(4 40 32 37)
(5 28 7 8)(9 46 58 80)(10 79 56 93)(11 49 25 14)(12 52 35 27)
(13 66 20 41) (16 71 63 89)(17 91 55 87)(18 38 47 67)(19 96 78 31)
(23 61 68 50)(29 48 59 94)(30 83 70 85)(34 82 36 90)(39 62 54 81)

(42 74 86 60)(43 92 65 73)(44 75 76 69)(45 95 84 57)(51 53 88 77)

are easily obtained from Table I. $g_1, g_2 \in G$ are the elements with the following permutation representations:

$g_1 =$ (1 11)(2 5)(3 13)(4 12)(6 25)(7 24)(8 21)(9 43)(10 19)(14 15)
(16 36)(17 38)(18 91)(20 26)(22 41)(23 69)(27 40)(28 64)(29 45)(30 54)
(31 79)(32 35)(33 49)(34 63)(37 52)(39 70)(42 53)(44 50)(46 73)(47 87)
(48 57)(51 74)(55 67)(56 78)(58 65)(59 84)(60 88)(61 76)(62 83)(66 72)
(68 75)(71 82)(77 86)(80 92)(81 85)(89 90)(93 96)(94 95),

$g_2 =$ (1 26)(2 32)(3 6)(4 24)(5 35)(7 12)(8 52)(9 59)(10 70)(11 20)
(13 25)(14 66)(15 72)(16 61)(17 86)(18 88)(19 39)(21 37)(22 33)(23 89)
(27 28)(29 58)(30 56)(31 81)(34 44)(36 76)(38 77)(40 64)(41 49)(42 55)
(43 84)(45 65)(46 94)(47 51)(48 80)(50 63)(53 67)(54 78)(57 92)(60 91)
(62 96)(68 71)(69 90)(73 95)(74 87)(75 82)(79 85)(83 93).

Given facts and Theorem 1.1 allow us to obtain difference set in each group H_i taking D as a starting structure. Namely, if P denotes any point from the set V of the points of D, the regularity of H_i-action on V insures

$$\{P^g \mid g \in H_i\} = V \quad \text{and} \quad g_1, g_2 \in H_i, g_1 \neq g_2 \Longleftrightarrow P^{g_1} \neq P^{g_2}, i = 1, \ldots, 5.$$

Thus, for each $i \in \{1, \ldots, 5\}$ we can first identify the elements of V with the elements of H_i, and then take as difference set Δ_i a subset of H_i corresponding to any D-block, for instance the base block (3). Different D-blocks lead to equivalent difference sets. One simple identification, determined by the choice of P, arises from the assignments

$$V \ni P \leftrightarrow \textit{identity element in } H_i \text{ and } V \ni P^g \leftrightarrow g \in H_i.$$

Taking now $P \equiv 1$, that is, the initial identification

$$1 \leftrightarrow \textit{identity element in } H_i \text{ (which we denote by 1 as well),}$$

then following the H_i-action on D (Table I) together with using the appropriate isomorphism $\omega_i^{-1} : G_i \to H_i$, we transcribe the base block (3) of D into inequivalent difference sets $\Delta_i \subseteq H_i, i = 1, \ldots, 5$.

$$\begin{aligned}
\Delta_{1,2,4} = \;& 1 + y^2 + z + z^2 + yz + y^3z^2 + zw + z^2w \\
&+x(z + yz^2 + y^2z^2w + y^3z^2w) \\
&+x^2(1 + y^2 + yzw + y^3z^2w) \\
&+x^3(z^2 + y^3z + yzw + y^2zw); \\
\Delta_3 = \;& 1 + y^2 + z + z^2 + yz^2w + y^3zw + zw + z^2w \\
&+x(z + y^3z + yzw + y^2z^2w) \\
&+x^2(1 + y^2 + yz^2 + y^3z) \\
&+x^3(z^2 + yz^2 + y^2zw + y^3z^2w); \\
\Delta_5 = \;& 1 + z + z^2 + zw + z^2w + zt + s + z^2ts \\
&+x(z + z^2t + zws + zwts) \\
&+x^2(1 + z^2wt + s + zwts) \\
&+x^3(z^2 + z^2wt + z^2ws + zts).
\end{aligned}$$

In a customary manner, they are denoted as the elements of the integral group ring $\mathsf{Z}H_i$, where
$H_i = \{x^p y^j z^k w^l \mid p, j = 0, \ldots, 3; k = 0, 1, 2; l = 0, 1\}, i = 1, \ldots, 4;$
$H_5 = \{x^p z^j w^k t^l s^m \mid p = 0, \ldots, 3; j = 0, 1, 2; k, l, m = 0, 1\}$. For the difference sets obtained the relations $dev\Delta_i \cong D$, $i = 1, \ldots, 5$, hold.

As compared to the construction of $\left(q^{d+1}(1+\frac{q^{d+1}-1}{q-1}), q^{d}\frac{q^{d+1}-1}{q-1}, q^{d}\frac{q^{d}-1}{q-1}\right)$ difference sets by McFarland and Dillon ([9], [5]), we point out that the center of none of the groups $H_i, i = 1, \ldots, 5$, contains elementary abelian subgroup E_{16} of order $q^{d+1} = 16$. Namely, $Z(H_1) \cong Z_4 \times Z_4, Z(H_2) \cong \{1\}, Z(H_3) \cong E_4, Z(H_4) \cong Z_2 \times Z_6, Z(H_5) \cong E_4$.

Three different groups appear as groups of the right multipliers of $\Delta_i, i = 1, \ldots, 5$. Denoting these groups by $M_i, i = 1, \ldots, 5$, on the grounds of Theorem 1.2 and using [6] we get:

$M_1 \cong Z_6, M_2 \cong Z_3, M_3 \cong Z_2, M_4 \cong Z_6$, and $M_5 \cong Z_2$.

References

[1] T. Beth, D. Jungnickel, and H. Lenz, Design theory, Cambridge University Press, 1999.

[2] A. Beutelspacher, Einführung in die endliche Geometrie I, Bibliographisches Institut, Mannheim-Wien-Zürich, 1985.

[3] A.E. Brouwer, J.H. Koolen and M.H. Klin, A root graph that is locally the line graph of the Petersen graph, Discrete Math. **264** (2003) 13-24.

[4] C.J. Colbourn and J.H. Dinitz, Eds., The CRC Handbook of Combinatorial Designs, CRC Press, New York, 1996; URL of the home page with new results is http://www.emba.uvm.edu/~dinitz/newresults

[5] J.F. Dillon, Variations on a scheme of McFarland for noncyclic difference sets, *J. Combin. Theory Ser. A* **40** (1985), 9-21.

[6] [GAP 99] The GAP Group, GAP - - - Groups, Algorithms, and Programming, Version 4.1; Aachen, St Andrews, 1999.

[7] A. Golemac and T. Vučičić, New difference sets in nonabelian groups of order 100, *Journal of Combinatorial Designs,* **9**, *(*2001), 424-434.

[8] Z. Janko, Coset enumeration in groups and constructions of symmetric designs, *Combinatorics* **90**, Elsevier Science Publishers, 1992, 275-277.

[9] R.L. McFarland, A family of difference sets in non-cyclic groups, *J. Combin. Theory Ser. A* **15** (1973), 1-10.

[10] B.D. McKay, Nauty user's guide (version 1.5), Technical Report TR-CS-90-02, Computer Science Department, Australian National University, 1990.

[11] T. Vučičić, New symmetric designs and nonabelian difference sets with parameters (100,45,20), *Journal of Combinatorial Designs,* **8**, *(*2000), 291-299.

[12] W.D. Wallis, Construction of strongly regular graphs using affine designs, *Bull. Austr. Math. Soc.* **4** (1971), 41-49.

University of Split, Dept. of mathematics, Teslina 12/III, 21 000 Split, Croatia
e-mail: golemac@mapmf.pmfst.hr
e-mail: mandic@mapmf.pmfst.hr
e-mail: vucicic@mapmf.pmfst.hr